Adjoint Weighting for Critical Systems with Continuous Energy Monte Carlo

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Abstract

Adjoint weighting is important for calculating parameters in reactor physics. A new method is developed to calculate adjoint-weighted quantities with continuous energy Monte Carlo. The method is applied to computing point-reactor kinetics parameters and estimating changes in reactivity from small perturbations. The results are benchmarked to 1D discrete ordinates, experimental data, and direct Monte Carlo calculations.
Outline

- Need for adjoint-weighting in Monte Carlo
- Method description
- Reactor kinetics
- Perturbation theory

Background
Monte Carlo Methods

• Direct simulation of radiation transport

• Advantages
  – High-fidelity geometric representation
  – Continuous energy and angle physics

• Weaknesses
  – Slow statistical convergence compared to discrete ordinates
  – Adjoint calculations difficult

Why are adjoint fluxes useful?

• Many reactor physics quantities are ratios of weighted integrals (kinetics parameters, etc.)

\[ \Lambda = \frac{\langle \psi^+ \frac{1}{\psi} \psi \rangle}{\langle \psi^+ F \psi \rangle} \]

• Adjoint flux is convenient weighting factor

\[ \langle \psi^+ A \psi \rangle = \langle \psi A^+ \psi^+ \rangle \]

• Adjoint flux corresponds to importance of radiation with respect to a “response function”
Computing Adjoint

- **Deterministic method process:**
  1. Invert sign of streaming operator
  2. Transpose scattering matrix
  3. Use standard solution techniques

- **Monte Carlo options:**
  - Invert radiation transport physics – very difficult in CE
  - Forward solution methods (weight window generator)

Response in Eigenvalue Problems

- **What is the response function for the k-eigenvalue transport equation?**

\[ H^\dagger \psi^\dagger = \frac{1}{k} F^\dagger \psi^\dagger \]

- **Iterated Fission Probability:**

*Consider a neutron introduced into a critical system at a location in phase space. The expected steady state neutron population resulting from that original progenitor neutron is defined as the iterated fission probability.*
Iterated Fission Probability

• Monte Carlo already follows a neutron and its progeny through successive generations

• Need to track information about its progenitor

• Can existing random walks of a $k$-eigenvalue calculation be used?

Related Work

• KENO eigenvalue contribution estimator

• MCNIC

• Nauchi & Kameyama reactor kinetics parameter calculations
Method Overview

Method Terminology

• Track neutron lineage through many generations.
Method Terminology

- Track neutron lineage through many generations.

![Diagram showing neutron lineage through generations]

- Asymptotic Population
  - Progenitor
  - Gen. 0
  - Gen. 1
  - Gen. 2
  - Gen. n
  - Original Generation
  - Latent Generations
  - Asymptotic Generation
Method Overview

- In original generation store contribution for each progenitor of index \( p \):
  - Ex. Fission Source Contribution: \( \omega_p = w_{0,p} \)
  - Ex. Track length flux: \( \omega_p = \sum_{\text{re } p} w_{0,p} d_t \)
- Progenitor index \( p \) inherited by all progeny.
- Tally asymptotic population in asymptotic generation:
  \[
  \pi_p = \sum_{\text{re } p} \nu \Sigma_f w d_t \\
  \text{Tally} = \sum_p \pi_p \omega_p
  \]
Reactor Kinetics

- Point reactor approximation for criticality excursion analysis:

\[
\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t)
\]

\[
\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t)
\]

- Need to know averaged point reactor parameters for a given system.

Reactor Kinetics Parameters

- Neutron generation time:

\[
\Lambda = \frac{\langle \psi^\dagger \frac{1}{v} \psi \rangle}{\langle \psi^\dagger F \psi \rangle}
\]

- Effective delayed neutron fraction:

\[
\beta_{\text{eff}} = \frac{\langle \psi^\dagger B \psi \rangle}{\langle \psi^\dagger F \psi \rangle}
\]

- Rossi-alpha:

\[
\alpha = -\frac{\langle \psi^\dagger B \psi \rangle}{\langle \psi^\dagger \frac{1}{v} \psi \rangle}
\]
Reactor Kinetics Tallies

- Adjoint-weighted neutron density:
  \[ \langle \psi^* \frac{1}{V} \psi \rangle = \frac{1}{W} \sum_p \pi_p \sum_{\tau} \frac{1}{\nu_{\tau}} w_{0,\tau} d_{\tau} \]

- Adjoint-weighted total fission source:
  \[ \langle \psi^* F \psi \rangle = \frac{1}{W} k \sum_p \pi_p w_{0,p} \]

- Adjoint-weighted delayed fission source:
  \[ \langle \psi^* B \psi \rangle = \frac{1}{W} k \sum_{p=\beta} \pi_p w_{0,p} \]

Multigroup Partisn

<table>
<thead>
<tr>
<th>#</th>
<th>G</th>
<th>Problem Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>Bare thermal slab, fuel/moderator mix</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>Reflected thermal slab, fuel + moderator</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>Bare fast slab</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>Reflected fast slab</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>Bare slab w/ intermediate spectrum</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>Bare fast sphere</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>Reflected fast sphere</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>Highly reflective slab</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>Subcritical bare fast slab (k = 0.78)</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>Supercritical bare fast slab (k = 1.14)</td>
</tr>
</tbody>
</table>
### Multigroup Partisn

**Lifetime Comparisons**

<table>
<thead>
<tr>
<th>#</th>
<th>Partisn</th>
<th>MCNP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.1323 µs</td>
<td>14.1025 +/- 0.0545 µs</td>
</tr>
<tr>
<td>2</td>
<td>135.2317 µs</td>
<td>135.0876 +/- 0.2081 µs</td>
</tr>
<tr>
<td>3</td>
<td>9.8100 ns</td>
<td>9.8099 +/- 0.0010 ns</td>
</tr>
<tr>
<td>4</td>
<td>43.4114 ns</td>
<td>43.5719 +/- 0.0913 ns</td>
</tr>
<tr>
<td>5</td>
<td>112.0523 ns</td>
<td>112.5003 +/- 0.4341 ns</td>
</tr>
<tr>
<td>6</td>
<td>1.7211 ns</td>
<td>1.7185 +/- 0.0022 ns</td>
</tr>
<tr>
<td>7</td>
<td>10.1982 ns</td>
<td>10.1969 +/- 0.0158 ns</td>
</tr>
<tr>
<td>8</td>
<td>6.1221 µs</td>
<td>6.1115 +/- 0.0073 µs</td>
</tr>
<tr>
<td>9</td>
<td>10.1715 ns</td>
<td>10.1714 +/- 0.0138 ns</td>
</tr>
<tr>
<td>10</td>
<td>9.6725 ns</td>
<td>9.6752 +/- 0.0115 ns</td>
</tr>
</tbody>
</table>

### Experimental Rossi-α

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Measured α (ms⁻¹)</th>
<th>ACODE α (ms⁻¹)</th>
<th>Progenitor α (ms⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Godiva</td>
<td>-1110 +/- 20</td>
<td>-1030 +/- 60</td>
<td>-1136 +/- 12</td>
</tr>
<tr>
<td>Jezebel</td>
<td>-640 +/- 10</td>
<td>-510 +/- 120</td>
<td>-643 +/- 13</td>
</tr>
<tr>
<td>Flattop-23</td>
<td>-267 +/- 5</td>
<td>-252 +/- 30</td>
<td>-296 +/- 5</td>
</tr>
<tr>
<td>BIG TEN</td>
<td>-117 +/- 1</td>
<td>-120 +/- 5</td>
<td>-122 +/- 2.5</td>
</tr>
<tr>
<td>STACY-29</td>
<td>-0.122 +/- 0.004</td>
<td>--</td>
<td>-0.128 +/- 0.002</td>
</tr>
<tr>
<td>WINCO-5</td>
<td>-1.1093 +/- 0.0003</td>
<td>--</td>
<td>-1.153 +/- 0.037</td>
</tr>
</tbody>
</table>

Note: Measured and ACODE are from experiment, Progenitor is the reference. All values are given in ms⁻¹.
Linear Perturbation

Perturbation Theory

- Suppose we want to know the change in reactivity due to a small change in a system
  - Density changes
  - Enrichment/concentration uncertainties
  - Cross section data changes

- Can use linear perturbation theory

\[
\Delta \rho = - \frac{\langle \psi^* P \psi \rangle}{\langle \psi^* F \psi \rangle}
\]

\[
P = \Delta \Sigma_i - \Delta S - \frac{1}{k} \Delta F
\]
Limitations in MCNP Perturbation

- MCNP perturbations do not account for fission source perturbations.
  - See talk by Jeff Favorite

- Linear perturbation theory can account for this

- Approach still has some limitations in this respect
  - Large local flux shifts cause problems

Reactor Kinetics Tallies

- Adjoint-weighted collision rate perturbation:
  \[ \langle \psi^\dagger \Delta \Sigma \psi \rangle = \frac{1}{W} \sum_p \pi_p \sum_{s \in P} \Sigma_s,_{s,} w_0,_{p,} d_s \]

- Adjoint-weighted scattering source perturbation:
  \[ \langle \psi^\dagger \Delta S \psi \rangle = \frac{1}{W} \sum_p \pi_p \sum_{s \in P} \Delta \Sigma_s,_{s,} w_0,_{p,} \]

- Adjoint-weighted fission source perturbation:
  \[ \langle \psi^\dagger \frac{1}{k} \Delta F \psi \rangle = \frac{1}{W} \sum_p \pi_p \sum_{f \in P} \frac{\Delta \nu \Sigma_f,_{f,}}{\nu \Sigma_f} w_0,_{p,} \]
Perturbation Validation

• Boron-10 worth in 2D APWR quarter core.

• $^{10}\text{B}$ concentration: $1.675 \times 10^{-4}$ to $1.65 \times 10^{-4}$

<table>
<thead>
<tr>
<th>$k_{\text{eff}}$</th>
<th>$0.99983 \pm 0.00008$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. $\Delta k$</td>
<td>$0.00325 \pm 0.00011$</td>
</tr>
<tr>
<td>Calc. $\Delta k$</td>
<td>$0.00320 \pm 0.00011$</td>
</tr>
</tbody>
</table>

Ref. obtained by comparing $k_{\text{eff}}$ for two separate MCNP runs.

Perturbation Validation

• Godiva $^{235}\text{U}$ sphere

• Change cross section library from ENDF/B-VI.5 to ENDF/B-VII.0

<table>
<thead>
<tr>
<th>$k_{\text{eff}}$</th>
<th>$0.99646 \pm 0.00004$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. $\Delta k$</td>
<td>$0.00344 \pm 0.00006$</td>
</tr>
<tr>
<td>Calc. $\Delta k$</td>
<td>$0.00358 \pm 0.00006$</td>
</tr>
</tbody>
</table>

Ref. obtained by comparing $k_{\text{eff}}$ for two separate MCNP runs.
Perturbation Validation

• Unit fuel cell with control rod

• Change enrichment by +0.01%

<table>
<thead>
<tr>
<th>$k_{\text{eff}}$</th>
<th>1.00390 +/- 0.00017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. $\Delta k$</td>
<td>0.00125 +/- 0.00025</td>
</tr>
<tr>
<td>Calc. $\Delta k$</td>
<td>0.00131 +/- 0.00025</td>
</tr>
</tbody>
</table>

Ref. obtained by comparing $k_{\text{eff}}$ for two separate MCNP runs.

Perturbation Validation

• Unit fuel cell with control rod

• No $^{131}\text{Xe}$ to 5 ppb $^{131}\text{Xe}$ in fuel

<table>
<thead>
<tr>
<th>$k_{\text{eff}}$</th>
<th>1.00368 +/- 0.00014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. $\Delta k$</td>
<td>-0.01687 +/- 0.00020</td>
</tr>
<tr>
<td>Calc. $\Delta k$</td>
<td>-0.01722 +/- 0.00020</td>
</tr>
</tbody>
</table>

Ref. obtained by comparing $k_{\text{eff}}$ for two separate MCNP runs.
Summary & Future Work

Summary

• Adjoint weighting is useful for calculating critical system parameters

• New method extends Monte Carlo to do adjoint-weighted tallies

• Applied to:
  – Kinetics parameters
  – Linear perturbations of reactivity
Future Work

- Further application to sensitivity/uncertainty analysis
- Subcritical source importance weighting

Questions?