Title: Tracking Charged Particles Through Magnetic Fields Using MCNP and MCNPX

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INTRODUCTION

The multi-particle Monte Carlo transport codes MCNP™ (Ref. 1) and MCNPX™ (Ref. 2) have been modified with a patch that allows specialized tracking of charged particles through the magnetic fields of a charged-particle beam optics system using pre-generated maps output from the COSY INFINITY code. A map is the rule for updating the particles’ phase space through a magnetic element. A file containing a single COSY map is assigned to each magnetic cell, which must be a vacuum. For current applications, the COSY maps are generated for protons, but any charged particle will be properly transported.

IMPLEMENTATION OF THE COSY MAP DATA

The output from the COSY INFINITY code is a map that determines the evolution of a particle’s canonical variables as the particle travels from one end of a magnetic cell to the other. The maps advance a particle’s canonical variables using a Taylor series expansion of the initial variables. Assuming that the particle is traveling along the MCNP/MCNPX z axis, the canonical variables, and their relationships to the MCNP/MCNPX variables, are: 

\[ x = \frac{xxx}{100}, \quad y = \frac{yyy}{100}, \]

\[ xp = \tan \left[ \arcsin \left( \frac{uuu}{\sqrt{uuu^2 + www^2}} \right) \right], \]  

\[ yp = \tan \left[ \arcsin \left( \frac{vvv}{\sqrt{vvv^2 + www^2}} \right) \right], \]
where the MCNP/MCNPX variables xxx and yyy are the incident particle’s horizontal and vertical position (cm), respectively; uuu, vvv, and www are the direction cosines of the particle’s trajectory; and erg is the particle’s kinetic energy (MeV). The parameter $E_{\text{tune}}$ is the nominal tune energy (MeV) of the magnet, i.e., the energy for which the map is calculated. (Another canonical variable, sometimes denoted $dl$, is unused here.)

The COSY map is a list of coefficients and exponents, each grouping of which comprises one term of the Taylor series expansion. The exponents are the power that each canonical variable is raised to in that term. For example, if the exponents corresponding to $x$ and $xp$ were 1 and 2, respectively (and the other exponents were zero), the canonical variable term would have $x \cdot xp^2$. The coefficients are then multiplied by this canonical variable term for each canonical variable and added for all the terms in the map.

In certain experiments, $E_{\text{tune}}$ is optimized for focusing to $E_{\text{tune, opt}}$ (near the nominal value), which is accounted for by adjusting the coefficients for the $dt$ terms (i.e., those for which the $idt$ exponent is nonzero) using

$$c_{\text{corrected}} = c_{\text{map}} \times \frac{\gamma_{\text{eff}} / (1 + \gamma_{\text{eff}})}{\gamma_{\text{map}} / (1 + \gamma_{\text{map}})} ,$$

where $\gamma_{\text{map}} = (E_{\text{tune}} + m_p) / m_p$, $\gamma_{\text{eff}} = (E_{\text{tune, opt}} + m_p) / m_p$, and $m_p$ is the mass (MeV) of the particle for which the map is generated (the “map particle”). The actual kinetic energy of map particles entering the map is accounted for by using $E_{\text{tune, opt}}$ for $E_{\text{tune}}$ in Eq. (3).
Tracking through these maps for arbitrary charged particles is achieved by assuming that the maps depend only upon the momentum and charge of the particle. Thus, for a particle of the same charge as the map particle, the map is applied to a map particle that has the same momentum as the incident particle, i.e. the energy (erg) in the \( dt \) term [Eq. (3)] is replaced by the “equivalent energy”

\[
E_{eq,x} = \left[ E_x \left( E_x + 2m_x \right) + m_p^2 \right]^{1/2} - m_p,
\]

where \( E_x \) and \( m_x \) are the kinetic energy and mass of the incident particle. For oppositely charged particles, the maps rotate 90° using a right hand rule (because of the \( q\vec{v} \times \vec{B} \) Lorentz force).

The beam pipe within a magnetic element is modeled as an elliptical cylinder. If the particle’s final position is beyond the elliptical boundary, the particle is assumed to have escaped the pipe and is killed. Since the position is checked only when the particle exits the element, escapes within the element are not modeled.

**APPLICATIONS**

The COSY map tracking feature has allowed the Monte Carlo simulation of proton radiography experiments from a pencil beam of protons incident on a diffuser, through conditioning magnets, an object, focusing elements, collimators, and then through the image planes. As an example of the utility of this capability, the effect of various types of collimators on the background noise appearing on image plates during a radiography experiment was studied.\(^6\) Only the quadrupole magnets used for focusing were modeled with COSY; all other elements were modeled as physical objects where detailed Monte Carlo transport was performed. Thus, collimator-beam interaction was modeled explicitly. Figure 1 shows the radiographed object and the first 100 proton
tracks, which clearly show the focusing effects of the magnets. This type of study would be difficult to perform deterministically, even without the difficulty of tracking correctly through the accelerator’s magnetic fields.
Figure 1. Perspective view, slightly off the beam axis, of the radiographed object (22.5 cm radius) and 100 proton tracks (9940 cm length along the axis). The wobbles in the proton tracks are caused by the COSY maps simulating the focusing effect of the quadrupole magnets. The diffuser and collimators are not shown.
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2. H. GRADY HUGHES, RICHARD E. PRAEL, and ROBERT C. LITTLE,
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5. The home page for COSY INFINITY is http://bt.nscl.msu.edu/cosy.
6 JEFFREY A. FAVORITE, “Collimator Study for the 933 Proton Radiography